Chapter 9. Indices

Ex 9.1

Answer 1.

(i) $6^{0} = 1$

(ii) $\left(\frac{1}{2}\right)^{-3} = (2)^{3} = 8$

(iii) $2^{3^{2}} = 2^{6} = 64$

(iv) $3^{2^{3}} = 3^{6} = 729$

(v) $(0.008)^{\frac{2}{3}} = \left(0.2^{3}\right)^{\frac{2}{3}} = (0.2)^{3\times\frac{2}{3}} = (0.2)^{2} = 0.04$

(vi) $(0.00243)^{\frac{-3}{5}} = \frac{1}{(0.00243)^{\frac{3}{5}}} = \frac{1}{\left(0.3^{5}\right)^{\frac{3}{5}}} = \frac{1}{(0.3)^{3}} = \frac{1}{0.027}$

(vii) $\sqrt[6]{25^{3}} = \sqrt[6]{\left(5^{2}\right)^{3}} = \sqrt[6]{5^{6}} = 5^{6\times\frac{1}{6}} = 5$

(viii) $\left(2\frac{10}{27}\right)^{\frac{2}{3}} = \left(\frac{64}{27}\right)^{\frac{2}{3}} = \left(\frac{4}{3}\right)^{3\times\frac{2}{3}} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}$

Answer 2A.

$$9^{4} \div 27^{-\frac{2}{3}} = \left[(3)^{2}\right]^{4} \div \left[(3)^{3}\right]^{-\frac{2}{3}}$$

$$= (3)^{2\times 4} \div (3)^{3\times\left(-\frac{2}{3}\right)} \quad .....\left(\text{Using } \left(a^{m}\right)^{n} = a^{mn}\right)$$

$$= (3)^{8} \div (3)^{-2}$$

$$= (3)^{8-(-2)} \quad .....\left(\text{Using } a^{m} \div a^{n} = a^{m-n}\right)$$

$$= (3)^{8+2}$$

$$= 3^{10}$$

$$= (3)^{2\times 5}$$

$$= \left[(3)^{2}\right]^{5}$$

$$= [9]^{5}$$

$$= 59049$$

Answer 2B.

$$7^{-4} \times (343)^{\frac{2}{3}} \div (49)^{-\frac{1}{2}}$$

$$= 7^{-4} \times (7^3)^{\frac{2}{3}} \div (7^2)^{-\frac{1}{2}}$$

$$= 7^{-4} \times 7^{3\times\frac{2}{3}} \div 7^{2\times\left(-\frac{1}{2}\right)}$$

$$= 7^{-4} \times 7^2 \div 7^{-1}$$

$$= 7^{-4+2-(-1)} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n}\right)$$

$$= 7^{-4+2+1}$$

$$= 7^{-1}$$

$$= \frac{1}{7} \quad \ldots\ldots\left(\text{Using } a^{-m} = \frac{1}{a^m}\right)$$

Answer 2C.

$$\left(\frac{64}{216}\right)^{\frac{2}{3}} \times \left(\frac{16}{36}\right)^{-\frac{3}{2}} = \left(\frac{2^6}{6^3}\right)^{\frac{2}{3}} \times \left(\frac{2^4}{6^2}\right)^{-\frac{3}{2}}$$

$$= \frac{(2^6)^{\frac{2}{3}}}{(6^3)^{\frac{2}{3}}} \times \frac{(2^4)^{-\frac{3}{2}}}{(6^2)^{-\frac{3}{2}}}$$

$$= \frac{(2)^{6\times\frac{2}{3}}}{(6)^{3\times\frac{2}{3}}} \times \frac{(2)^{4\times\left(-\frac{3}{2}\right)}}{(6)^{2\times\left(-\frac{3}{2}\right)}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \frac{(2)^{2\times2}}{(6)^2} \times \frac{(2)^{2\times(-3)}}{(6)^{-3}}$$

$$= \frac{(2)^4}{(6)^2} \times \frac{(2)^{-6}}{(6)^{-3}}$$

$$= \frac{(2)^4}{(6)^2} \times \frac{(6)^3}{(2)^6} \quad \ldots\ldots\left(\text{Using } a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{(2)^4}{(2)^6} \times \frac{(6)^3}{(6)^2}$$

$$= (2)^{4-6} \times (6)^{3-2} \quad \ldots\ldots\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= (2)^{-2} \times (6)^1$$

$$= \frac{1}{2^2} \times 6$$

$$= \frac{1}{4} \times 6$$

$$= \frac{3}{2}$$

Answer 3A.

$$\left(a^3\right)^5 \times a^4 = (a)^{3\times5} \times a^4 \quad .....\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= (a)^{15} \times a^4$$

$$= a^{15+4} \quad .....\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= a^{19}$$

Answer 3B.

$$a^2 \times a^3 \div a^4 = a^{2+3-4} \quad .....\left(\text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n}\right)$$

$$= a^1$$

$$= a$$

Answer 3C.

$$a^{\frac{1}{3}} \div a^{-\frac{2}{3}} = a^{\frac{1}{3}-\left(-\frac{2}{3}\right)} \quad .....\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= a^{\frac{1}{3}+\frac{2}{3}}$$

$$= a^{\frac{1+2}{3}}$$

$$= a^1$$

$$= a$$

Answer 3D.

$$a^{-3} \times a^2 \times a^0 = a^{-3+2+0} \quad .....\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= a^{-1}$$

$$= \frac{1}{a}$$

Answer 3E.

$$\left(b^{-2} - a^{-2}\right) \div \left(b^{-1} - a^{-1}\right) = a^{-3+2+0} \quad .....\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= a^{-1}$$

$$= \frac{1}{a}$$

Answer 4.

(i) $\dfrac{2^3 \times 3^5 \times 24^2}{12^2 \times 18^3 \times 27}$

$$= \frac{2^3 \times 3^5 \times \left(2^3 \times 3\right)^2}{\left(2^2 \times 3\right)^2 \times \left(2 \times 3^2\right)^3 \times \left(3^3\right)}$$

$$= \frac{2^3 \times 3^5 \times 2^6 \times 3^2}{2^4 \times 3^2 \times 2^3 \times 3^6 \times 3^3}$$

$$= \frac{2^3 \times 3^5 \times 2^6 \times 3^2}{2^4 \times 3^2 \times 2^3 \times 3^6 \times 3^3}$$

$$= \frac{2^9 \times 3^7}{2^7 \times 3^{11}} = \frac{2^{9-7}}{3^{11-7}} = \frac{2^2}{3^4} = \frac{4}{81}$$

(ii) $\dfrac{4^3 \times 3^7 \times 5^6}{5^8 \times 2^7 \times 3^3}$

$$= \frac{\left(2^2\right)^3 \times 3^{7-3}}{5^{8-6} \times 2^7}$$

$$= \frac{2^6 \times 3^4}{5^2 \times 2^7}$$

$$= \frac{3^4}{5^2 \times 2^{7-6}} = \frac{81}{5^2 \times 2^1} = \frac{81}{50}$$

(iii) $\dfrac{12^2 \times 75^{-2} \times 35 \times 400}{48^2 \times 15^{-3} \times 525}$

$$= \frac{\left(2^2 \times 3\right)^2 \times (7 \times 5) \times \left(2^4 \times 5^2\right) \times (3 \times 5)^3}{\left(2^4 \times 3\right)^2 \times \left(3 \times 5^2 \times 7\right) \times \left(3 \times 5^2\right)^2}$$

$$= \frac{2^4 \times 3^2 \times 7 \times 5 \times 2^4 \times 5^2 \times 3^3 \times 5^3}{2^8 \times 3^2 \times 3 \times 5^2 \times 7 \times 3^2 \times 5^4}$$

$$= \frac{2^{4+4} \times 3^{2+3} \times 5^{1+2+3} \times 7}{2^8 \times 3^{2+1+2} \times 5^{4+2} \times 7}$$

$$= \frac{2^8 \times 3^5 \times 5^6 \times 7}{2^8 \times 3^5 \times 5^6 \times 7}$$

$=1$

(iv) $\dfrac{2^6 \times 5^{-4} \times 3^{-3} \times 4^2}{8^3 \times 15^{-3} \times 25^{-1}}$

$$= \frac{2^6 \times \left(2^2\right)^2 \times (3 \times 5)^3 \times \left(5^2\right)^1}{\left(2^3\right)^3 \times 5^4 \times 3^3}$$

$$= \frac{2^{6+4} \times 3^3 \times 5^{3+2}}{2^9 \times 3^3 \times 5^4} = 2^{10-9} \times 5^{5-4} = 2 \times 5 = 10$$

Answer 5A.

$$3p^{-2}q^{3} \div 2p^{3}q^{-2} = \frac{3p^{-2}q^{3}}{2p^{3}q^{-2}}$$
$$= \frac{3}{2}\left[\frac{p^{-2}}{p^{3}} \times \frac{q^{3}}{q^{-2}}\right]$$
$$= \frac{3}{2}\left[\left(p^{-2} \div p^{3}\right) \times \left(q^{3} \div q^{-2}\right)\right]$$
$$= \frac{3}{2}\left[\left(p^{-2-3}\right) \times \left(q^{3-(-2)}\right)\right] \quad .....\left(\text{Using } a^{m} \div a^{n} = a^{m-n}\right)$$
$$= \frac{3}{2}\left[\left(p^{-5}\right) \times \left(q^{5}\right)\right]$$
$$= \frac{3}{2}\left[\left(\frac{1}{p^{5}}\right) \times \left(q^{5}\right)\right]$$
$$= \frac{3q^{5}}{2p^{5}}$$

Answer 5B.

$$\left[\left(p^{-3}\right)^{\frac{2}{3}}\right]^{\frac{1}{2}} = p^{-3 \times \frac{2}{3} \times \frac{1}{2}} \quad .....\left(\text{Using } \left(a^{m}\right)^{n} = a^{mn}\right)$$
$$= p^{-1}$$
$$= \frac{1}{p}$$

Answer 6.

(i) $\left(1 - \frac{15}{64}\right)^{-\frac{1}{2}} = \left(\frac{64 - 15}{64}\right)^{\frac{-1}{2}} = \left(\frac{49}{64}\right)^{\frac{-1}{2}} = \left(\frac{64}{49}\right)^{\frac{1}{2}} = \frac{8}{7}$

(ii) $\left(\frac{8}{27}\right)^{\frac{-2}{3}} - \left(\frac{1}{3}\right)^{-2} - 7^{0}$

$$= \left(\frac{27}{8}\right)^{\frac{2}{3}} - (3)^{2} - 1$$
$$= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - 9 - 1$$
$$= \left(\frac{3}{2}\right)^{2} - 10$$
$$= \frac{9}{4} - 10 = \frac{9 - 40}{4} = \frac{-31}{4}$$

(iii) $9^{\frac{5}{2}} - 3 \times 5^{0} - \left(\frac{1}{81}\right)^{\frac{-1}{2}}$

$= 3^{2 \times \frac{5}{2}} - 3 \times 1 - \left(\frac{1}{81}\right)^{\frac{-1}{2}}$

$= 3^{5} - 3 - 9^{2 \times \frac{1}{2}}$

$= 243 - 3 - 9$

$= 231$

(iv) $(27)^{\frac{2}{3}} \times 8^{\frac{-1}{6}} \div 18^{\frac{-1}{2}}$

$= 3^{3 \times \frac{2}{3}} \times \frac{1}{2^{3 \times \frac{1}{6}}} \div \left(\frac{1}{18}\right)^{\frac{1}{2}}$

$= \frac{3^{2}}{2^{\frac{1}{2}}} \times \left(2 \times 3^{2}\right)^{\frac{1}{2}}$

$= \frac{3^{2}}{2^{\frac{1}{2}}} \times 2^{\frac{1}{2}} \times 3$

$= 3^{2+1} = 3^{3} = 27$

(v) $16^{\frac{3}{4}} + 2\left(\frac{1}{2}\right)^{-1} \times 3^{0}$

$= 2^{4 \times \frac{3}{4}} + 2 \times 2 \times 1$

$= 2^{3} + 4$

$= 2^{3} + 4 = 8 + 4 = 12$

(vi) $\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$

$= \left(\frac{1}{2^{2}}\right)^{\frac{1}{2}} + (0.1)^{-1} - 3^{2}$

$= \frac{1}{2} + (0.1)^{-1} - 3^{2}$

$= \frac{1}{2} + \frac{1}{0.1} - 9$

$= \frac{1}{2} + \frac{10}{1} - 9$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$

Answer 7A.

$$\left(27x^9\right)^{\frac{2}{3}} = \left(3^3x^9\right)^{\frac{2}{3}}$$

$$= \left(3^3\right)^{\frac{2}{3}}\left(x^9\right)^{\frac{2}{3}} \quad .....\left(\text{Using } (a\times b)^n = a^n \times b^n\right)$$

$$= (3)^{3\times\frac{2}{3}}(x)^{9\times\frac{2}{3}} \quad .....\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= (3)^2 x^{3\times 2}$$

$$= 9\ x^6$$

Answer 7B.

$$(8x^6y^3)^{\frac{2}{3}} = (2^3x^6y^3)^{\frac{2}{3}}$$

$$= (2^3)^{\frac{2}{3}}(x^6)^{\frac{2}{3}}(y^3)^{\frac{2}{3}} \quad .....\left(\text{Using } (a\times b)^n = a^n \times b^n\right)$$

$$= (2)^{3\times\frac{2}{3}}(x)^{6\times\frac{2}{3}}(y)^{3\times\frac{2}{3}} \quad .....\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= (2)^2(x)^4(y)^2$$

$$= 4x^4y^2$$

Answer 7C.

$$\left(\frac{64a^{12}}{27b^6}\right)^{-\frac{2}{3}} = \left(\frac{2^6a^{12}}{3^3b^6}\right)^{-\frac{2}{3}}$$

$$= \left(\frac{2^{6\times\left(-\frac{2}{3}\right)}a^{12\times\left(-\frac{2}{3}\right)}}{3^{3\times\left(-\frac{2}{3}\right)}b^{6\times\left(-\frac{2}{3}\right)}}\right) \quad ....\left(\text{Using } (a\times b)^n = a^n \times b^n \text{ and } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right)$$

$$= \frac{2^{-4}a^{-8}}{3^{-2}b^{-4}}$$

$$= \frac{3^2b^4}{2^4a^8} \quad .....\left(\text{Using } a^{-n} = \frac{1}{a^n}\right)$$

$$= \frac{9b^4}{16a^8}$$

Answer 7D.

$$\left(\frac{36m^{-4}}{49n^{-2}}\right)^{-\frac{3}{2}} = \left(\frac{6^2m^{-4}}{7^2n^{-2}}\right)^{-\frac{3}{2}}$$

$$= \left(\frac{6^{2\times\left(-\frac{3}{2}\right)}m^{-4\times\left(-\frac{3}{2}\right)}}{7^{2\times\left(-\frac{3}{2}\right)}n^{-2\times\left(-\frac{3}{2}\right)}}\right) \quad \ldots\ldots\left(\text{Using } (a\times b)^n = a^n \times b^n \text{ and } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right)$$

$$= \frac{6^{-3}m^6}{7^{-3}n^3}$$

$$= \frac{7^3m^6}{6^3n^3} \quad \ldots\ldots\left(\text{Using } a^{-n} = \frac{1}{a^n}\right)$$

$$= \frac{343m^6}{216n^3}$$

Answer 7E.

$$\left(a^{\frac{1}{3}} + a^{-\frac{1}{3}}\right)\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right)$$

$$= a^{\frac{1}{3}}\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right) + a^{-\frac{1}{3}}\left(a^{\frac{2}{3}} - 1 + a^{-\frac{2}{3}}\right)$$

$$= \left(a^{\frac{1}{3}} \times a^{\frac{2}{3}} - a^{\frac{1}{3}} \times 1 + a^{\frac{1}{3}} \times a^{-\frac{2}{3}}\right) + \left(a^{-\frac{1}{3}} \times a^{\frac{2}{3}} - a^{-\frac{1}{3}} \times 1 + a^{-\frac{1}{3}} \times a^{-\frac{2}{3}}\right)$$

$$= \left(a^{\frac{1}{3}+\frac{2}{3}} - a^{\frac{1}{3}} \times 1 + a^{\frac{1}{3}-\frac{2}{3}}\right) + \left(a^{-\frac{1}{3}+\frac{2}{3}} - a^{-\frac{1}{3}} + a^{-\frac{1}{3}-\frac{2}{3}}\right)$$

$$\ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= \left(a^1 - a^{\frac{1}{3}} + a^{-\frac{1}{3}}\right) + \left(a^{\frac{1}{3}} - a^{-\frac{1}{3}} + a^{-1}\right)$$

$$= a - a^{\frac{1}{3}} + a^{-\frac{1}{3}} + a^{\frac{1}{3}} - a^{-\frac{1}{3}} + \frac{1}{a}$$

$$= a + \frac{1}{a}$$

Answer 7F.

$$\sqrt[3]{x^4y^2} \div \sqrt[6]{x^5y^{-5}}$$

$$= \left(x^4y^2\right)^{\frac{1}{3}} \div \left(x^5y^{-5}\right)^{\frac{1}{6}}$$

$$= \left(x^{4\times\frac{1}{3}}y^{2\times\frac{1}{3}}\right) \div \left(x^{5\times\frac{1}{6}}y^{-5\times\frac{1}{6}}\right) \quad .....\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \left(x^{\frac{4}{3}}y^{\frac{2}{3}}\right) \div \left(x^{\frac{5}{6}}y^{-\frac{5}{6}}\right)$$

$$= \frac{x^{\frac{4}{3}}y^{\frac{2}{3}}}{x^{\frac{5}{6}}y^{-\frac{5}{6}}}$$

$$= x^{\frac{4}{3}-\frac{5}{6}}\ y^{\frac{2}{3}-\left(-\frac{5}{6}\right)} \quad .....\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= x^{\frac{1}{2}}\ y^{\frac{3}{2}}$$

$$= x^{\frac{1}{2}}\left(y^3\right)^{\frac{1}{2}} \quad .....\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \sqrt{x}\sqrt{y^3}$$

$$= \sqrt{xy^3}$$

Answer 7G.

$$\left\{\left(a^m\right)^{m-\frac{1}{m}}\right\}^{\frac{1}{m+1}} = (a)^{m\times\left(m-\frac{1}{m}\right)\times\left(\frac{1}{m+1}\right)} \quad .....\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$\text{Consider, } m\times\left(m-\frac{1}{m}\right)\times\left(\frac{1}{m+1}\right)$$

$$= \left(m^2-1\right)\times\left(\frac{1}{m+1}\right)$$

$$= m^2\times\left(\frac{1}{m+1}\right) - 1\times\left(\frac{1}{m+1}\right)$$

$$= \frac{m^2}{m+1} - \frac{1}{m+1}$$

$$= \frac{m^2-1}{m+1}$$

$$= \frac{(m-1)(m+1)}{m+1}$$

$$= m-1$$

$$(a)^{m\times\left(m-\frac{1}{m}\right)\times\left(\frac{1}{m+1}\right)} = a^{m-1}$$

Answer 7H.

$$x^{m+2n} \cdot x^{3m-8n} \div x^{5m-60}$$

$$= x^{m+2n+3m-8n-5m-(-60)} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n} \text{ and } a^m \div a^n = a^{m-n}\right)$$

$$= x^{m+2n+3m-8n-5m+60}$$

$$= x^{-m-6n+60}$$

Answer 7I.

$$(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{\frac{-2}{5}} + 8^{\frac{1}{3}} \cdot \left(\frac{1}{2}\right)^{-1} \cdot 2^0$$

$$= \left(3^4\right)^{\frac{3}{4}} - \left(\frac{1}{2^5}\right)^{\frac{-2}{5}} + \left(2^3\right)^{\frac{1}{3}} \cdot \left(\frac{1}{2}\right)^{-1} \times 1 \quad \ldots\ldots\left(\text{Using } a^0 = 1\right)$$

$$= 3^{4\times\frac{3}{4}} - \frac{1}{2^{5\times\left(\frac{-2}{5}\right)}} + 2^{3\times\frac{1}{3}} \cdot (2)^1 \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= 3^3 - \frac{1}{2^{-2}} + 2^1 \cdot (2)^1$$

$$= 3^3 - 2^2 + 2^{1+1} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n} \text{ and } a^{-n} = \frac{1}{a^n}\right)$$

$$= 3^3 - 2^2 + 2^2$$

$$= 27$$

Answer 7J.

$$\left(\frac{27}{343}\right)^{\frac{2}{3}} \div \frac{1}{\left(\frac{625}{1296}\right)^{\frac{1}{4}}} \times \frac{536}{\sqrt[3]{27}}$$

$$= \left(\frac{3^3}{7^3}\right)^{\frac{2}{3}} \div \frac{1}{\left(\frac{5^4}{2^4 \times 3^4}\right)^{\frac{1}{4}}} \times \frac{2^3 \times 67}{\sqrt[3]{3^3}}$$

$$= \left(\frac{3^3}{7^3}\right)^{\frac{2}{3}} \div \frac{1}{\left(\frac{5^4}{2^4 \times 3^4}\right)^{\frac{1}{4}}} \times \frac{2^3 \times 67}{\left(3^3\right)^{\frac{1}{3}}}$$

$$= \left(\frac{3^{3\times\frac{2}{3}}}{7^{3\times\frac{2}{3}}}\right) \div \frac{1}{\left(\frac{5^{4\times\frac{1}{4}}}{2^{4\times\frac{1}{4}} \times 3^{4\times\frac{1}{4}}}\right)} \times \frac{2^3 \times 67}{3^{3\times\frac{1}{3}}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \left(\frac{3^2}{7^2}\right) \div \frac{1}{\left(\frac{5^1}{2^1 \times 3^1}\right)} \times \frac{2^3 \times 67}{3^1}$$

$$= \left(\frac{3^2}{7^2}\right) \times \left(\frac{5^1}{2^1 \times 3^1}\right) \times \left(\frac{2^3 \times 67}{3^1}\right)$$
$$= 3^{2-1-1} \times 2^{3-1} \times 5^1 \times 7^2 \times 67$$
$$= 3^0 \times 2^2 \times 5^1 \times 7^2 \times 67$$
$$= 1 \times 4 \times 5 \times 49 \times 67 \qquad \ldots\ldots(\text{Using } a^0 = 1)$$
$$= 65660$$

Answer 8.

(i) $$\frac{5^x \times 7 - 5^x}{5^{x+2} - 5^{x+1}} = \frac{5^x(7-1)}{5^{x+1}(5-1)}$$
$$= \frac{5^{x-x-1} \times 6}{4}$$
$$= \frac{5^{-1} \times 6}{4}$$
$$= \frac{6}{5 \times 4} = \frac{3}{10}$$

(ii) $$\frac{3^{x+1} + 3^x}{3^{x+3} - 3^{x+1}} = \frac{3^x(3+1)}{3^x\left(3^3 - 3\right)}$$
$$= \frac{4}{27-3} = \frac{4}{24} = \frac{1}{6}$$

(iii) $$\frac{2^m \times 3 - 2^m}{2^{m+4} - 2^{m+1}} = \frac{2^m(3-1)}{2^m\left(2^4 - 2\right)}$$
$$= \frac{2}{16-2} = \frac{2}{14} = \frac{1}{7}$$

(iv) $$\frac{5^{n+2} - 6.5^{n+1}}{13.5^n - 2.5^{n+1}} = \frac{5^n\left(5^2 - 6 \times 5\right)}{5^n(13 - 2 \times 5)} = \frac{25-30}{13-10} = \frac{-5}{3}$$

Answer 9A.

$$2^{2x+1} = 8$$
$$\Rightarrow 2^{2x+1} = 2^3$$
$$\Rightarrow 2x + 1 = 3$$
$$\Rightarrow 2x = 2$$
$$\Rightarrow x = 1$$

Answer 9B.

$$3 \times 7^x = 7 \times 3^x$$
$$\Rightarrow \frac{7^x}{7} = \frac{3^x}{3}$$
$$\Rightarrow 7^{x-1} = 3^{x-1} \quad .....\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$
$$\Rightarrow 7^{x-1} = 3^{x-1} \times 1$$
$$\Rightarrow 7^{x-1} = 3^{x-1} \times 7^0 \quad ......(\text{Using } a^0 = 1)$$
$$\Rightarrow x - 1 = 0$$
$$\Rightarrow x = 1$$

Answer 9C.

$$2^{x+3} + 2^{x+1} = 320$$
$$\Rightarrow 2^{x+3} + 2^{x+1} = 2^6 \times 5$$
$$\Rightarrow 2^x \cdot 2^3 + 2^x \cdot 2^1 = 2^6 \times 5$$
$$\Rightarrow 2^x\left(2^3 + 2^1\right) = 2^6 \times 5$$
$$\Rightarrow 2^x(8 + 2) = 2^6 \times 5$$
$$\Rightarrow 2^x(10) = 2^6 \times 5$$
$$\Rightarrow 2^x\left(\frac{10}{5}\right) = 2^6$$
$$\Rightarrow 2^x \cdot 2 = 2^6$$
$$\Rightarrow \frac{2^x \cdot 2}{2^6} = 1$$
$$\Rightarrow 2^{x+1-6} = 1 \times 2^0$$
$$\Rightarrow 2^{x-5} = 1 \times 2^0$$
$$\Rightarrow x - 5 = 0$$
$$\Rightarrow x = 5$$

Answer 9D.

$$9 \times 3^x = (27)^{2x-5}$$
$$\Rightarrow 3^2 \times 3^x = \left(3^3\right)^{2x-5}$$
$$\Rightarrow 3^2 \times 3^x = 3^{3(2x-5)}$$
$$\Rightarrow 3^{2+x} = 3^{6x-15}$$
$$\Rightarrow 1 = \frac{3^{6x-15}}{3^{2+x}}$$
$$\Rightarrow 1 = 3^{6x-15-2-x}$$
$$\Rightarrow 3^0 = 3^{5x-17}$$
$$\Rightarrow 5x - 17 = 0$$
$$\Rightarrow x = \frac{17}{5}$$

Answer 9E.

$$2^{2x+3} - 9 \times 2^x + 1 = 0$$
$$2^{2x} \cdot 2^3 - 9 \times 2^x + 1 = 0$$
$$\text{Put } 2^x = t, \text{ so, } 2^{2x} = t^2$$
$$\text{So, } 2^{2x} \cdot 2^3 - 9 \times 2^x + 1 = 0 \text{ becomes } 8t^2 - 9t + 1 = 0$$
$$\Rightarrow 8t^2 - 8t - t + 1 = 0$$
$$\Rightarrow 8t(t-1) - (t-1) = 0$$
$$\Rightarrow t - 1 = 0 \text{ or } 8t - 1 = 0$$
$$\Rightarrow t = 1 \text{ or } t = \frac{1}{8}$$
$$\Rightarrow 2^x = 1 \text{ or } 2^x = \frac{1}{2^3}$$
$$\Rightarrow 2^x = 2^0 \text{ or } 2^x = 2^{-3}$$
$$\Rightarrow x = 0 \text{ or } x = -3$$

Answer 9F.

$$1 = p^x$$
$$\Rightarrow p^0 = p^x \quad \text{......(Using } a^0 = 1)$$
$$\Rightarrow x = 0$$

Answer 9G.

$$p^3 \times p^{-2} = p^x$$
$$\Rightarrow p^{3+(-2)} = p^x \quad \text{.....}\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$
$$\Rightarrow p^1 = p^x$$
$$\Rightarrow x = 1$$

Answer 9H.

$$p^{-5} = \frac{1}{p^{x+1}}$$
$$\Rightarrow p^{-5} \times p^{x+1} = 1$$
$$\Rightarrow p^{-5+x+1} = 1$$
$$\Rightarrow p^{x-4} = p^0$$
$$\Rightarrow x - 4 = 0$$
$$\Rightarrow x = 4$$

Answer 9I.

$$2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$$
$$\Rightarrow 2^{2x} + 2^{x+2} - 2^2 \times 2^3 = 0$$
$$\Rightarrow 2^{2x} + 2^x \cdot 2^2 - 2^{2+3} = 0 \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$
$$\Rightarrow 2^{2x} + 2^x \cdot 2^2 - 2^5 = 0$$
$$\Rightarrow 2^{2x} + 2^x \cdot 4 - 32 = 0$$

Put $2^x = t$

So, $2^{2x} = t^2$

$2^{2x} + 2^{x+2} - 32 = 0$ becomes $t^2 + 4t - 32 = 0$

$$\Rightarrow (t+8)(t-4) = 0$$
$$\Rightarrow t+8 = 0 \quad \text{or} \quad t-4 = 0$$
$$\Rightarrow t = -8 \quad \text{or} \quad t = 4$$
$$\Rightarrow 2^x = -8 \quad \text{or} \quad 2^x = 4$$
$$\Rightarrow 2^x = -2^3 \quad \text{or} \quad 2^x = 2^2$$

Using the second equation $2^x = 2^2$, we get $x = 2$.

Answer 9J.

$$9 \times 81^x = \frac{1}{27^{x-3}}$$
$$\Rightarrow 3^2 \times 3^{4x} = \frac{1}{3^{3(x-3)}}$$
$$\Rightarrow 3^2 \times 3^{4x} = \frac{1}{3^{3x-9}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$
$$\Rightarrow 3^2 \times 3^{4x} \times 3^{3x-9} = 1$$
$$\Rightarrow 3^{2+4+3x-9} = 1 \times 3^0$$
$$\Rightarrow 2 + 4 + 3x - 9 = 0$$
$$\Rightarrow 3x - 3 = 0$$
$$\Rightarrow x = 1$$

Answer 9K.

$$2^{2x-1} - 9 \times 2^{x-2} + 1 = 0$$
$$2^{2x} \cdot 2^{-1} - 9 \times 2^{x} \cdot 2^{-2} + 1 = 0$$
Let $2^x = t$, so $2^{2x} = t^2$

So, $2^{2x} \cdot 2^{-1} - 9 \times 2^{x} \cdot 2^{-2} + 1 = 0$ becomes $\frac{t^2}{2} - 9 \times \frac{t}{2^2} + 1 = 0$

$$\Rightarrow \frac{t^2}{2} - \frac{9t}{4} + 1 = 0$$
$$\Rightarrow 2t^2 - 9t + 4 = 0$$
$$\Rightarrow 2t^2 - 8t - t + 4 = 0$$
$$\Rightarrow 2t(t-4) - 1(t-4) = 0$$
$$\Rightarrow (t-4)(2t-1) = 0$$
$$\Rightarrow t - 4 = 0 \text{ or } 2t - 1 = 0$$
$$\Rightarrow t = 4 \text{ or } t = \frac{1}{2}$$
$$\text{So, } 2^x = 4 \text{ or } 2^x = \frac{1}{2}$$
$$\Rightarrow 2^x = 2^2 \text{ or } 2^x = 2^{-1}$$
$$\Rightarrow x = 2 \text{ or } x = -1$$

Answer 9L.

$$5^{x^2} : 5^x = 25 : 1$$
$$\Rightarrow \frac{5^{x^2}}{5^x} = \frac{25}{1}$$
$$\Rightarrow \frac{5^{x^2}}{5^x} = \frac{5^2}{1}$$
$$\Rightarrow 5^{x^2} = 5^2 \times 5^x$$
$$\Rightarrow 5^{x^2} = 5^{2+x}$$
$$\Rightarrow x^2 = 2 + x$$
$$\Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow (x-2)(x+1) = 0$$
$$\Rightarrow x - 2 = 0 \text{ or } x + 1 = 0$$
$$\Rightarrow x = 2 \text{ or } x = -1$$

Answer 9M.

$$\sqrt{\left(8^0+\frac{2}{3}\right)}=(0.6)^{2-3x}$$

$$\Rightarrow\left(1+\frac{2}{3}\right)^{\frac{1}{2}}=\left(\frac{6}{10}\right)^{2-3x}$$

$$\Rightarrow\left(\frac{5}{3}\right)^{\frac{1}{2}}=\left(\frac{3}{5}\right)^{2-3x}$$

$$\Rightarrow\left(\frac{3}{5}\right)^{-\frac{1}{2}}=\left(\frac{3}{5}\right)^{2-3x}$$

$$\Rightarrow-\frac{1}{2}=2-3x$$

$$\Rightarrow-1=4-6x$$

$$\Rightarrow-5=-6x$$

$$\Rightarrow x=\frac{5}{6}$$

Answer 9N.

$$\sqrt{\left(\frac{3}{5}\right)^{x+3}}=\frac{27^{-1}}{125^{-1}}$$

$$\Rightarrow\left(\frac{3}{5}\right)^{(x+3)\times\left(\frac{1}{2}\right)}=\frac{\left(3^3\right)^{-1}}{\left(5^3\right)^{-1}}$$

$$\Rightarrow\left(\frac{3}{5}\right)^{\frac{x+3}{2}}=\left(\frac{3}{5}\right)^{-3}$$

$$\Rightarrow\frac{x+3}{2}=-3$$

$$\Rightarrow x+3=-6$$

$$\Rightarrow x=-9$$

Answer 9O.

$$9^{x+4}=3^2\times(27)^{x+1}$$

$$\Rightarrow 9^{x+4}=3^2\times\left(3^3\right)^{x+1}$$

$$\Rightarrow 3^{2(x+4)}=3^2\times3^{3x+3}$$

$$\Rightarrow 3^{2x+8}=3^{2+3x+3}$$

$$\Rightarrow 2x+8=2+3x+3$$

$$\Rightarrow 2x+8=3x+5$$

$$\Rightarrow x=3$$

Answer 10.

(i) $\left(\sqrt[3]{8}\right)^{\frac{-1}{2}} = 2^k$

$$\Rightarrow 8^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$
$$\Rightarrow \left(2^3\right)^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$
$$\Rightarrow \left(2^3\right)^{\frac{1}{3} \times \frac{-1}{2}} = 2^k$$
$$\Rightarrow 2^{\frac{-1}{2}} = 2^k$$
$$= k = -\frac{1}{2}$$

(ii) $\sqrt[4]{\sqrt[3]{x^2}} = x^k$

$$\Rightarrow \left\{\left(x^2\right)^{\frac{1}{3}}\right\}^{\frac{1}{4}} = x^k$$
$$\Rightarrow \left(x^2\right)^{\frac{1}{12}} = x^k$$
$$\Rightarrow x^{\frac{2}{12}} = x^k$$
$$\Rightarrow x^{\frac{1}{6}} = x^k$$
$$\Rightarrow k = \frac{1}{6}$$

(iii) $\left(\sqrt{9}\right)^{-7} \times \left(\sqrt{3}\right)^{-5} = 3^k$

$$\Rightarrow \left\{\left(3^2\right)^{\frac{1}{2}}\right\}^{-7} \left\{(3)^{\frac{1}{2}}\right\}^{-5} = 3^k$$
$$\Rightarrow 3^{-7} \times 3^{\frac{-5}{2}} = 3^k$$
$$\Rightarrow 3^{-7-\frac{5}{2}} = 3^k$$
$$\Rightarrow 3^{\frac{-14-5}{2}} = 3^k$$
$$\Rightarrow 3^{\frac{-19}{2}} = 3^k$$
$$= k = \frac{-19}{2}$$

(iv) $\left(\frac{1}{3}\right)^{-4} \div 9^{\frac{-1}{3}} = 3^k$

$$\Rightarrow \left(3^{-1}\right)^{-4} \div \left(3^2\right)^{\frac{-1}{3}} = 3^k$$
$$\Rightarrow 3^4 \div 3^{\frac{-2}{3}} = 3^k$$
$$\Rightarrow 3^{4+\frac{2}{3}} = 3^k$$
$$\Rightarrow 3^{\frac{14}{3}} = 3^k$$
$$\Rightarrow k = \frac{14}{3}$$

Answer 11.

$$a = 2^{\frac{1}{3}} - 2^{\frac{-1}{3}}$$

$$\Rightarrow a = 2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}}$$

$$\Rightarrow a^3 = \left(2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}}\right)^3 = 2 - \frac{1}{2} - 3\left(2^{\frac{1}{3}} - \frac{1}{2^{\frac{1}{3}}}\right)$$

$$\Rightarrow a^3 = \frac{4-1}{2} - 3a$$

$$\Rightarrow a^3 = \frac{3}{2} - 3a$$

$$\Rightarrow 2a^3 + 6a = 3$$

Answer 12.

$$x = 3^{\frac{2}{3}} + 3^{\frac{1}{3}}$$

$$\Rightarrow x^3 = 3^2 + 3 + 3 \times 3^{\frac{2}{3}} \times 3^{\frac{1}{3}}\left(3^{\frac{2}{3}} + 3^{\frac{1}{3}}\right)$$

$$\Rightarrow x^3 = 9 + 3 + 3 \times 3^{\frac{2}{3}+\frac{1}{3}}(x)$$

$$\Rightarrow x^3 = 12 + 9x$$

$$\Rightarrow x^3 - 9x - 12 = 0$$

Answer 13.

Let $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$

$$\Rightarrow a^{\frac{1}{x}} = k,\ b^{\frac{1}{y}} = k,\ c^{\frac{1}{z}} = k$$

$$\Rightarrow a = k^x,\ b = k^y,\ c = k^z$$

It is also given that $abc = 1$

$$\Rightarrow k^x \times k^y \times k^z = 1$$

$$\Rightarrow k^{x+y+z} = k^0$$

$$\Rightarrow x + y + z = 0$$

Answer 14.

Let $a^x = b^y = c^z = k$

$$\Rightarrow a = k^{\frac{1}{x}},\ b = k^{\frac{1}{y}},\ c = k^{\frac{1}{z}}$$

It is also given that $b^2 = ac$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow y = \frac{2zx}{z+x}$$

Answer 15.

$$\text{LHS} = \frac{1}{1+a^{p-q}} + \frac{1}{1+a^{q-p}}$$

$$= \frac{1+a^{q-p}+1+a^{p-q}}{\left(1+a^{p-q}\right)\left(1+a^{q-p}\right)}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{\left(1+a^{p-q}\right)\left(1+a^{-(p-q)}\right)}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{1+a^{-(p-q)}+a^{p-q}+a^{p-q}\cdot a^{-(p-q)}}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{1+a^{-(p-q)}+a^{p-q}+a^{p-q-p+q}}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{1+a^{-(p-q)}+a^{p-q}+a^{0}}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{1+a^{-(p-q)}+a^{p-q}+1}$$

$$= \frac{2+a^{-(p-q)}+a^{p-q}}{2+a^{-(p-q)}+a^{p-q}}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

Answer 16.

$$9^{p+2} - 9^p = 240$$
$$\Rightarrow 9^p\left(9^2 - 1\right) = 240$$
$$\Rightarrow 9^p(80) = 240$$
$$\Rightarrow 9^p = 3$$
$$\Rightarrow 3^{2p} = 3$$
$$\Rightarrow 2p = 1$$
$$\Rightarrow p = \frac{1}{2}$$
$$(8p)^p = \left(2^3p\right)^p$$
$$= \left(2^3 \cdot \frac{1}{2}\right)^{\frac{1}{2}}$$
$$= \left(2^{3-1}\right)^{\frac{1}{2}}$$
$$= \left(2^2\right)^{\frac{1}{2}}$$
$$= 2$$

Answer 17.

$$a^x = b^y = c^z$$
$$\text{So, } a^x = b^y \Rightarrow a = b^{\frac{y}{x}} \quad \ldots.. \left(\text{Using } a^{\frac{1}{n}} = \sqrt[n]{a}\right)$$
$$b^y = c^z \Rightarrow c = b^{\frac{y}{z}} \quad \ldots.. \left(\text{Using } a^{\frac{1}{n}} = \sqrt[n]{a}\right)$$
$$\text{and } abc = 1$$
$$\Rightarrow b^{\frac{y}{x}} \cdot b \cdot b^{\frac{y}{z}} = 1$$
$$\Rightarrow b^{\frac{y}{x}} \cdot b \cdot b^{\frac{y}{z}} = 1$$
$$\Rightarrow b^{\frac{y}{x}+1+\frac{y}{z}} = 1$$
$$\Rightarrow b^{\frac{y}{x}+1+\frac{y}{z}} = b^0 \quad \ldots.. \left(\text{Using } a^0 = 1\right)$$
$$\Rightarrow \frac{y}{x} + 1 + \frac{y}{z} = 0$$

Divide throughout by y.

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{y}{z} = 0$$

Hence proved.

Answer 18.

$$x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$$

$$\Rightarrow \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right) + z^{\frac{1}{3}} = 0 \text{ cubing both sides, we get :}$$

$$\Rightarrow \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^3 + z + 3\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right) z^{\frac{1}{3}} \left(x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}}\right) = 0$$

$$\Rightarrow x + y + 3\, x^{\frac{1}{x}} y^{\frac{1}{3}} \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right) + z + 0 = 0$$

$$\Rightarrow x + y + 3\, x^{\frac{1}{3}} y^{\frac{1}{3}} \left(-z^{\frac{1}{3}}\right) + z = 0 \qquad \text{(Using the given condition again)}$$

$$\Rightarrow x + y + z = 3\, x^{\frac{1}{3}}\ y^{\frac{1}{3}}\ z^{\frac{1}{3}}$$

$$\Rightarrow (x + y + z)^3 = 27xyz$$

Answer 19.

$$\text{Given } 2250 = 2^a \cdot 3^b \cdot 5^c$$

$$\Rightarrow 3^2 \times 5^3 \times 2 = 2^a \cdot 3^b \cdot 5$$

$$\Rightarrow a = 1, b = 2, c = 3$$

$$3^a \times 2^{-b} \times 5^{-c}$$

$$= 3^1 \times 2^{-2} \times 5^{-3}$$

$$= \frac{3}{2^2 \times 5^3}$$

$$= \frac{3}{500}$$

Answer 20.

$$2400 = 2^x \times 3^y \times 5^z$$

$$2400 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5$$

$$\therefore 2^x \times 3y \times 5^z = 2^5 \times 3^1 \times 5^2$$

$$\Rightarrow x = 5, y = 1, z = 2$$

$$\therefore 2^{-x} \times 3^y \times 5^z = 2^{-5} \times 3^1 \times 5^2$$

$$= \frac{1}{32} \times 3 \times 25 = \frac{75}{32}$$

Answer 21.

$$\text{Let } 2^x = 3^y = 12^z = k$$

$$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 12 = k^{\frac{1}{z}}$$

$$\text{Now, } 12 = 2 \times 2 \times 3$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x}} \times k^{\frac{1}{x}} \times k^{\frac{1}{y}}$$

$$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{x} + \frac{1}{y}$$

$$\Rightarrow \frac{1}{z} = \frac{2}{x} + \frac{1}{y}$$

Answer 22A.

$$9^{2a} = \left(\sqrt[3]{81}\right)^{\frac{-6}{b}} = \left(\sqrt{27}\right)^2$$

$$\Rightarrow 9^{2a} = \left(\sqrt[3]{3^4}\right)^{\frac{-6}{b}} = \left(\sqrt{3^3}\right)^2$$

$$\Rightarrow \left(3^2\right)^{2a} = \left(3^{4\times\frac{1}{3}}\right)^{\frac{-6}{b}} = \left(3^{3\times\frac{1}{2}}\right)^2$$

$$\Rightarrow 3^{4a} = \left(3^1\right)^{\frac{-8}{b}} = \left(3^1\right)^3$$

$$\Rightarrow 3^{4a} = \frac{-8}{b} = 3$$

$$\Rightarrow 3^{4a} = 3 \text{ and } \frac{-8}{b} = 3$$

$$\Rightarrow 3^{4a} = 3 \text{ and } \frac{-8}{b} = 3$$

$$\Rightarrow 4a = 3 \text{ and } b = \frac{-8}{3}$$

$$\Rightarrow a = \frac{3}{4} \text{ and } b = \frac{-8}{3}$$

Answer 22B.

$$(\sqrt{243})^a \div 3^{b+1} = 1 \text{ and } 27^b - 81^{4-\frac{a}{2}} = 0$$

$$\Rightarrow \left(\sqrt{3^5}\right)^a \div 3^{b+1} = 1 \text{ and } \left(3^3\right)^b - \left(3^4\right)^{4-\frac{a}{2}} = 0$$

$$\Rightarrow \left(3^5\right)^{\frac{a}{2}} \div 3^{b+1} = 1 \text{ and } 3^{3b} - \left(3^4\right)^{4-\frac{a}{2}} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}\right)} \div 3^{b+1} = 1 \text{ and } 3^{(3b)} - 3^{4\left(4-\frac{a}{2}\right)} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}-b-1\right)} = 1 \text{ and } 3^{(3b)} - 3^{16-2a} = 0$$

$$\Rightarrow 3^{\left(\frac{5a}{2}-b-1\right)} = 3^0 \text{ and } 3^{3b} = 3^{16-2a}$$

$$\Rightarrow \frac{5a}{2} - b - 1 = 0 \text{ and } 3b = 16 - 2a$$

$$\Rightarrow \frac{5a}{2} - b = 1 \text{ and } 2a + 3b = 16$$

$$\Rightarrow 5a - 2b = 2 \text{ and } 2a + 3b = 16$$

Multiply the equations by 3 and 2 respectively.

$\Rightarrow 15a - 6b = 6$ and $4a + 6b = 32$

Adding the equations,

$19a = 38$

$\Rightarrow a = 2$

Substitute the value of a in $5a - 2b = 2$ to find b.

$5a - 2b = 2$

$\Rightarrow 5(2) - 2b = 2$

$\Rightarrow 10 - 2b = 2$

$\Rightarrow b = 4$

Hence, $a = 2$ and $b = 4$.

Answer 23A.

$$\text{LHS} = \sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$$

$$= \sqrt{\frac{y}{x}} \cdot \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{x}{z}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \sqrt{\left(\frac{y}{x}\right)\left(\frac{z}{y}\right)\left(\frac{x}{z}\right)}$$

$$= \sqrt{x^{1-1} \cdot y^{1-1} \cdot z^{1-1}}$$

$$= \sqrt{x^0 \cdot y^0 \cdot z^0}$$

$$= \sqrt{1 \cdot 1 \cdot 1}$$

$$= 1 \quad \ldots\ldots(\text{Using } a^0 = 1)$$

$$= \text{RHS}$$

Hence proved.

Answer 23B.

$$\text{LHS} = \left(\frac{a^m}{a^n}\right)^{m+n-1} \cdot \left(\frac{a^n}{a^1}\right)^{n+1-m} \cdot \left(\frac{a^1}{a^m}\right)^{1+m-n}$$

$$= \frac{a^{m(m+n-1)}}{a^{n(m+n-1)}} \cdot \frac{a^{n(n+1-m)}}{a^{1(n+1-m)}} \cdot \frac{a^{1(1+m-n)}}{a^{m(1+m-n)}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \frac{a^{m^2+mn-m}}{a^{n^2+mn-n}} \cdot \frac{a^{n^2-mn+n}}{a^{n+1-m}} \cdot \frac{a^{1+m-n}}{a^{m^2-mn+m}}$$

$$= a^{m^2+mn-m-\left(n^2+mn-n\right)} \cdot a^{n^2-mn+n-(n+1-m)} \cdot a^{1+m-n-\left(m^2-mn+m\right)} \quad \ldots\ldots\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= a^{m^2+mn-m-n^2-mn+n} \cdot a^{n^2-mn+n-n-1+m} \cdot a^{1+m-n-m^2+mn-m}$$

$$= a^{m^2+mn-m-n^2-mn+n+n^2-mn+n-n-1+m+1+m-n-m^2+mn-m} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= a^0$$

$$= 1 \quad \ldots\ldots(\text{Using } a^0 = 1)$$

$$= \text{RHS}$$

Hence proved.

Answer 23C.

$$\text{LHS} = \left(\frac{a^m}{a^n}\right)^{m+n-1} \cdot \left(\frac{a^n}{a^1}\right)^{n+1-m} \cdot \left(\frac{a^1}{a^m}\right)^{1+m-n}$$

$$= \frac{a^{m(m+n-1)}}{a^{n(m+n-1)}} \cdot \frac{a^{n(n+1-m)}}{a^{1(n+1-m)}} \cdot \frac{a^{1(1+m-n)}}{a^{m(1+m-n)}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \frac{a^{m^2+mn-m}}{a^{n^2+mn-n}} \cdot \frac{a^{n^2-mn+n}}{a^{n+1-m}} \cdot \frac{a^{1+m-n}}{a^{m^2-mn+m}}$$

$$= a^{m^2+mn-m-\left(n^2+mn-n\right)} . a^{n^2-mn+n-(n+1-m)} \cdot a^{1+m-n-\left(m^2-mn+m\right)} \quad \ldots\ldots\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= a^{m^2+mn-m-n^2-mn+n} . a^{n^2-mn+n-n-1+m} \cdot a^{1+m-n-m^2+mn-m}$$

$$= a^{m^2+mn-m-n^2-mn+n+n^2-mn+n-n-1+m+1+m-n-m^2+mn-m} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= a^0$$

$$= 1 \qquad \ldots\ldots\left(\text{Using } a^0 = 1\right)$$

$$= \text{RHS}$$

Hence proved.

Answer 23D.

$$\text{LHS} = \sqrt[ab]{\frac{x^a}{x^b}} \cdot \sqrt[bc]{\frac{x^b}{x^c}} \cdot \sqrt[ca]{\frac{x^c}{x^a}}$$

$$= \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$$

$$= \frac{x^{\frac{1}{b}}}{x^{\frac{1}{a}}} \cdot \frac{x^{\frac{1}{c}}}{x^{\frac{1}{b}}} \cdot \frac{x^{\frac{1}{a}}}{x^{\frac{1}{c}}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= x^{\frac{1}{b}-\frac{1}{a}} . x^{\frac{1}{c}-\frac{1}{b}} . x^{\frac{1}{a}-\frac{1}{c}} \quad \ldots\ldots\left(\text{Using } a^m \div a^n = a^{m-n}\right)$$

$$= x^{\frac{a-b}{ab}} . x^{\frac{b-c}{bc}} . x^{\frac{c-a}{ac}}$$

$$= x^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ac}} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= x^{\frac{ac-bc+ab-ac+bc-ab}{abc}}$$

$$= x^{\frac{0}{abc}}$$

$$= x^0$$

$$= 1 \qquad \ldots\ldots\left(\text{Using } a^0 = 1\right)$$

$$= \text{RHS}$$

Hence proved.

Answer 23E.

$$\text{LHS} = \left(x^a\right)^{b-c} \times \left(x^b\right)^{c-a} \times \left(x^c\right)^{a-b}$$

$$= x^{a(b-c)} \times x^{b(c-a)} \times x^{c(a-b)} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= x^{ab-ac} \times x^{bc-ab} \times x^{ac-bc}$$

$$= x^{ab-ac+bc-ab+ac-bc} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= x^0$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

Answer 23F.

$$\text{LHS} = \frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r$$

$$= \frac{x^{p(q-r)}}{x^{q(p-r)}} \div \frac{x^{qr}}{x^{pr}} \quad \ldots\ldots\left(\text{Using } \left(a^m\right)^n = a^{mn}\right)$$

$$= \frac{x^{p(q-r)}}{x^{q(p-r)}} \times \frac{x^{pr}}{x^{qr}}$$

$$= \frac{x^{pq-pr}}{x^{pq-qr}} \times \frac{x^{pr}}{x^{qr}}$$

$$= \frac{x^{pq-pr+pr}}{x^{pq-qr+qr}} \quad \ldots\ldots\left(\text{Using } a^m \times a^n = a^{m+n}\right)$$

$$= \frac{x^{pq}}{x^{pq}}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.